# GCSE Maths - Algebra

## Solving Linear Equations

Worksheet

NOTES

SOLUTIONS

This worksheet will show you how to work out different types of questions involving linear equations. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

# Section A

## Worked Example

**Solve the equation $4x + 9 = -3$**

**Step 1:** Rearrange the equation to ensure only terms containing the unknown are present on the left-hand side.

*We want to rearrange the equation so that only terms containing $x$ are present on the left. This means any whole number present on the left-hand side should be eliminated.*

*Subtract $9$ from both sides of the equation:*

$$4x + 9 = -3$$
$$4x = -12$$

**Step 2:** Eliminate the coefficient of the unknown by dividing both sides of the equation by the coefficient of the unknown variable.

*Divide both sides by $4$:*

$$4x = -12$$
$$x = -3$$

**Step 3:** Check the answer by substituting the value of the unknown back into the original equation. The value of the left-hand side should be equal to the value of the right-hand side.

*Substitute $x = -3$:*

$$4x + 9 = 4(-3) + 9 = -3$$
$$-3 = -3$$

*Hence, the final answer is $\boldsymbol{x = -3}$.*

## Guided Example

**Solve the equation $5 - 2y = -11$**

**Step 1:** Rearrange the equation to ensure only terms containing the unknown are present on the left-hand side.

**Step 2:** Eliminate the coefficient of the unknown by dividing both sides of the equation by the coefficient of the unknown variable.

**Step 3:** Check the answer by substituting the value of the unknown back into the original equation.

**Now it's your turn!**

If you get stuck, look back at the worked and guided examples.

1. Solve the following linear equations for $x$:

a) $3x - 4 = 11$

b) $4 - 8x = 20$

c) $10 - x = 4$

d) $2x + 9 = 7$

# Section B

## Worked Example

**Solve the equation $5x - 12 = 3x + 4$**

**Step 1:** Rearrange the equation to ensure only terms containing the unknown are present on the left-hand side.

$$5x - 12 = 3x + 4$$

*Subtract $3x$ from both sides of the equation:*

$$5x - 12 - 3x = 3x + 4 - 3x$$

$$2x - 12 = 4$$

*Eliminate the constant term on the same side as the unknown.*

$$2x - 12 = 4$$

*Add $12$ to both sides of the equation:*

$$2x - 12 + 12 = 4 + 12$$

$$2x = 16$$

**Step 2:** Eliminate the coefficient of the unknown by dividing both sides of the equation by the coefficient of the unknown variable.

$$2x = 16$$

*Divide both sides of the equation by $2$:*

$$x = 8$$

**Step 3:** Check the answer by substituting the value of the unknown back into the original equation.

$$5x - 12 = 3x + 4$$

$$5(8) - 12 = 3(8) + 4$$

$$28 = 28$$

*Hence, the final answer is $\boldsymbol{x = 8}$.*

## Guided Example

**Solve the equation $3x - 15 = 7x + 5$**

**Step 1:** Rearrange the equation to ensure only terms containing the unknown are present on the left-hand side.

**Step 2:** Eliminate the coefficient of the unknown by dividing both sides of the equation by the coefficient of the unknown variable.

**Step 3:** Check the answer by substituting the value of the unknown back into the original equation.

**Now it's your turn!**

If you get stuck, look back at the worked and guided examples.

2. Solve the following linear equations for $x$:

a) $2 + 5x = 6 - 3x$

b) $2x - 15 = 5x - 21$

c) $5x + 15 = 4x + 5$

d) $-8 - 9x = 28 - 3x$

# Section C

## Worked Example

**Solve the equation $x - 1 = \frac{1}{4}(2x + 14)$**

**Step 1:** Expand any brackets present in the equation.

$$x - 1 = \frac{1}{4}(2x + 14)$$

$$x - 1 = \frac{2x + 14}{4}$$

**Step 2:** Eliminate any fractions by multiplying both sides of the equation by the greatest common denominator. Expand any brackets that appear.

*Here, the common denominator is $4$ so we multiply both sides by $4$:*

$$4(x - 1) = 4\left(\frac{2x + 14}{4}\right)$$
$$4x - 4 = 2x + 14$$

**Step 3:** Rearrange the equation to ensure only terms containing the unknown are present on the left-hand side.

$$4x - 4 = 2x + 14$$

*Subtract $2x$ from both sides of the equation:*

$$2x - 4 = 14$$

*Add $4$ to both sides of the equation to eliminate the constant term on the left:*

$$2x = 18$$

**Step 4:** Eliminate the coefficient of the unknown by dividing both sides of the equation by the coefficient of the unknown variable.

*Divide both sides of the equation by $2$:*

$$2x = 18$$
$$x = 9$$

**Step 5:** Check the answer by substituting the value of the unknown back into the original equation.

$$x - 1 = \frac{1}{4}(2x + 14)$$
$$9 - 1 = \frac{1}{4}(2(9) + 14)$$
$$8 = 8$$

*Hence, the final answer is $x = 9$.*

## Guided Example

**Solve the equation** $\frac{3}{2}(3x - 2) = 3x + 5$

**Step 1:** Expand any brackets present in the equation.

**Step 2:** Eliminate any fractions by multiplying both sides of the equation by the greatest common denominator. Expand any brackets that appear.

**Step 3:** Rearrange the equation to ensure only terms containing the unknown are present on the left-hand side.

**Step 4:** Eliminate the coefficient of the unknown by dividing both sides of the equation by the coefficient of the unknown variable.

**Step 5:** Check the answer by substituting the value of the unknown back into the original equation.

**Now it's your turn!**

If you get stuck, look back at the worked and guided examples.

3. Solve the following linear equations for $x$:

a) $\frac{1}{5}x = x - 2$

b) $2x - 6 = \frac{2}{3}x + 4$

c) $\frac{(4+3x)}{5} - \frac{(x+4)}{2} = \frac{3}{2}$

d) $\frac{(x-3)}{3} + \frac{(2x+4)}{2} = 5$

# Section D

## Worked Example

**A graph has equation $y = 5x + 4$.**
**Using a graph, determine the approximate value of $x$ when $y = 2$.**

**Step 1:** If the graph is not given, we need to sketch the graph. We can do this by determining the $x$ –intercept and the $y$ –intercept.

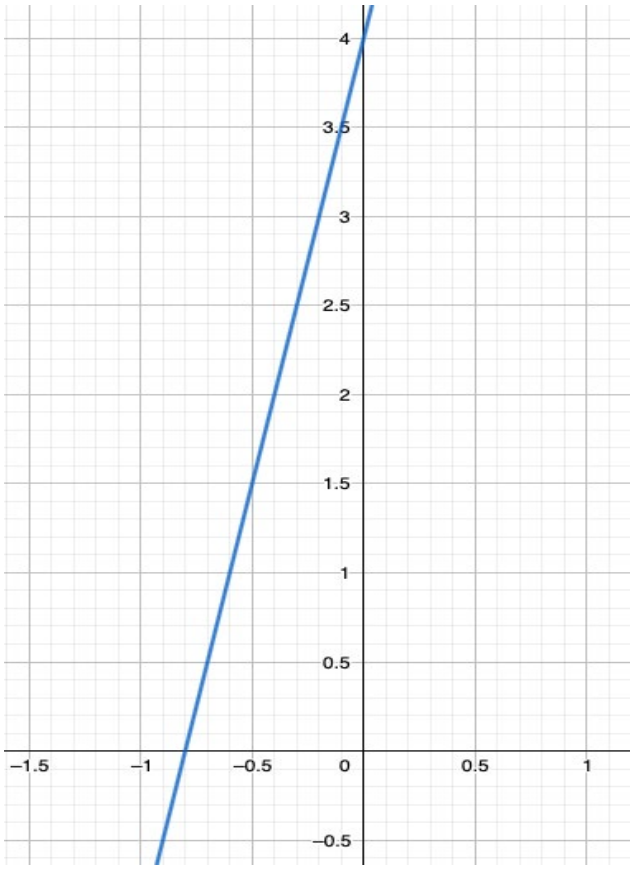
*With the use of the normal form of a straight-line equation, $y = mx + c$, where $m$ is the gradient and $c$ is the $y$ –intercept. We can determine both values from the given equation:*

$$y = mx + c$$
$$y = 5x + 4$$

*The gradient of the line is $m = 5$ and the $y$ –intercept is $c = 4$.*
*To determine the $x$ –intercept, substitute in $y = 0$:*

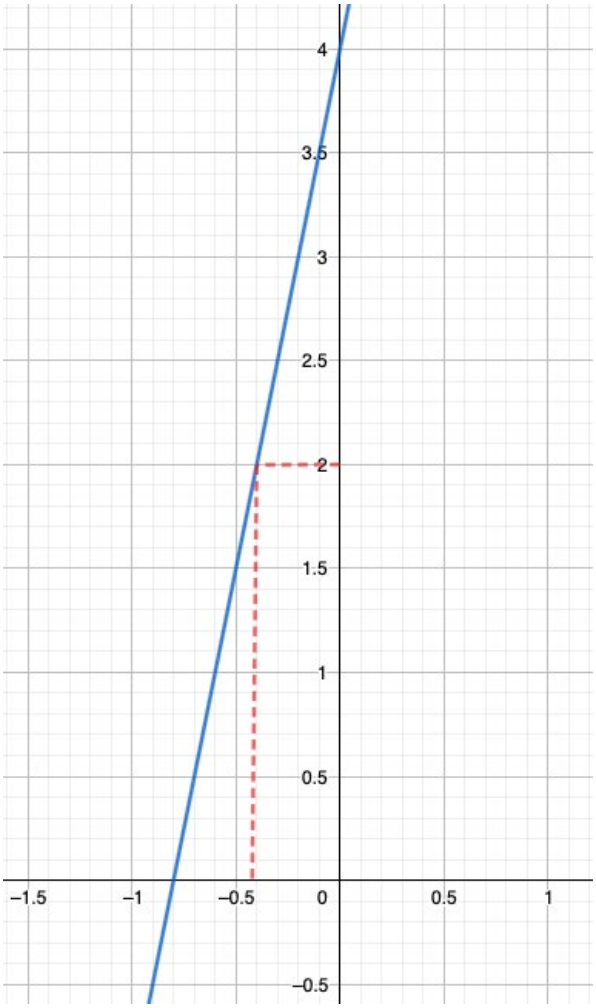
$$0 = 5x + 4$$
$$5x = -4$$
$$x = -\frac{4}{5}$$

**Step 2:** Using the coordinates of the $y$ –intercept and the $x$ –intercept, sketch the linear graph.



**Step 3:** Draw a dotted line on the graph from the value of $y$ that is required by the question to find the approximate value of $x$.

*In this case, since the question asks for the value of $x$ when $y = 2$, we draw a dotted line from $y = 2$.*




**Step 4:** Determine the approximate value of $x$ from the dotted line.

*From the graph, we identify where the red dotted line intersects the x-axis. This corresponds*

$$x = -0.4$$

*Hence, the approximate value of $x$ when $y = 2$ is $\boldsymbol{x = -0.4}$.*

## Guided Example

**A graph has equation $y = 3 - 2x$.**
**Using a graph, determine the approximate value of $x$ when $y = 2$.**

**Step 1:** If the graph is not given, we need to sketch the graph. We can do this by first determining the $x$ –intercept and the $y$ –intercept.

**Step 2:** Using the coordinates of the $y$ –intercept and the $x$ –intercept, sketch the linear graph.

**Step 3:** Draw a dotted line on the graph from the value of $y$ that is required by the question to find the approximate value of $x$.

**Step 4:** Determine the approximate value of $x$ from the dotted line.

**Now it's your turn!**

If you get stuck, look back at the worked and guided examples.

4. With the use of a graph, perform the following estimations:

a) The equation of a line is given by $y = 3x + \frac{1}{2}$.
Estimate the value of $y$ when $x = 1$.

b) The equation of the line is given by $y - 6 = 2 - 4x$.
Estimate the value of $x$ when $y = 2$.